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GENERALIZATION OF COMPLEMENTARITY*

1. Introduction

The *Principle of Complementarity* was introduced into physics by Niels Bohr in order to overcome certain conceptual difficulties.[1] Bohr suggested himself that complementarity might also be profitably applied to other fields[2]; however, he never attempted to show in sufficient detail how such a generalization of his principle can be accomplished.

The extension of complementarity from physics to other fields presupposes a general definition of this concept.[3] This definition has been implicitly presupposed in the application of complementarity to psychology by Brody and Oppenheim (1969), but Bohr's principle is often applied on an *ad hoc* basis in a variety of fields, without even an implicit general definition. Thus, without a formal generalization, 'complementarity' can serve as a convenient tool of bridging any contradictory theories or approaches without detailed analysis and without consideration to what kinds of problems the principle can or cannot solve.

It is the main purpose of this paper to give an *explicit general definition of complementarity*, compatible with Bohr's understanding of his principle and, as far as possible, free of technicalities. This paper will therefore be devoted to formulating and explicating the general definition as well as discussing the general relation of complementarity to epistemological dilemmas.

2. The assignment paradox

2.1. The generalization of a principle that has been formulated in a particular context presupposes first of all a *generalization of the problem* that this principle may solve. Therefore, the first, and at times tedious, task will be to attempt such a generalization of the problem.

Anticipating our results, we can say that the Principle of Complementarity is, roughly speaking, the definition of a 'complementarity' relation between certain complex entities for the purpose of removing

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what we call 'assignment paradoxes'. Assignment paradoxes are exemplified by perplexing assignments as the following: "the electron behaves sometimes 'corpuscularly' and sometimes 'wavelike'" (cf. Bedau and Oppenheim, 1961), or "hunger is sometimes 'mental' and sometimes 'bodily'" (cf. Brody and Oppenheim, 1969). The assignment paradoxes have in common that in each of them an expectation is violated. This expectation is generated by the *intended* use of particular characteristics and it is violated by the *de facto* use of these characteristics. We will deal with the formal side of this aspect in some detail, for only then will we be able to define the general problem as well as the general 'complementarity' relation.

The attempt to generalize the Principle of Complementarity as Bohr formulated it (more or less indirectly) necessitates that we abstract from a variety of specific features which were pertinent to the application of this principle in physics and also in psychology. Thus, the reader will be asked to endure a rather extensive introduction of terms and concepts only few of which have hitherto been connected with complementarity.

2.2. *Preliminary Definitions*

2.2.1. The term *entity* will be used to refer to thing, process, occurrence, and the like. We shall feel free to use this term to cover what may be different logical categories in other contexts.

2.2.2. An entity x is *equivalent* for person P to an entity y *with respect to a characteristic Ch* if and only if the following condition holds: whenever P assigns Ch to x he would have also assigned it to y if y had been in the place of x. Each entity is, of course, equivalent to itself.

For example, for P all automobiles may be equivalent with respect to the characteristic 'mechanical' but not all automobiles may be equivalent for him with respect to 'fast'. Which entities are equivalent with respect to a Ch and a person P is an empirical question. In exceptional cases an entity might be only equivalent to itself, but usually it will be the case that, if P assigns a characteristic to a particular entity, he thereby assigns it also to a number of other entities, given his assignments are not capricious.

Whenever the Ch and P are clearly implied in the context, we will not explicitly refer to them in order to relativize 'equivalent'.

2.3. *Characteristics and Their Assignments*

2.3.1. In the following paragraphs, we will make some important distinctions as to characteristics and their assignments, all of which are relativized to a person *P*. In addition, we presuppose that *P* is conscious of what characteristics he assigns to what entities, and that he is in many ways 'rational'. In particular, we assume that the characteristics he assigns are sufficiently precise[4] for him to allow us to ignore the possibility of capricious and 'erroneous' assignments. We also stipulate that if two characteristics are mutually exclusive for *P*, he will indeed never assign them at the same time to the same entity. Many concepts we define with regard to *P* are thus 'quasi-psychological'[5] rather than psychological.

2.3.2. Two characteristics, *Ch′* and *Ch″*, will be called *mutually exclusive* for *P* if and only if (a) *P* cannot conceive of assigning both at the same time to one and the same entity and (b) he can conceive of assigning both at the same time to some different entities.

The mutual exclusiveness can be logical, or established by a theory that *P* holds, or an entirely idiosyncratic belief of *P*; in any case, it is assumed to be stable for *P*.

2.3.3. An assignment of a characteristic *Ch* to an entity *x* by *P* will be called *de facto permanent* for *P* if and only if *P* never revokes the assignment with regard to *x* and entities equivalent to *x*.[6]

Let us take an example in order to illustrate the definition. If somebody assigns 'human' to a two-legged featherless animal it will, in all likelihood, be a de facto permanent assignment. Once he has assigned it to an entity, he will not revoke the assignment by, for instance, assigning to it (or equivalent entities) a characteristic which is mutually exclusive with 'human'.

2.3.4. A characteristic *Ch* will be called *intentionally permanent* for *P* if and only if *P* expects every one of its assignments to be a de facto permanent assignment.

For most people, 'male' is an intentionally permanent characteristic in that *each* of its assignments carries the expectation that it will never be revoked. The expectation itself may be based on an explicit theory that *P* holds, or on *P*'s intuition, or *P*'s idiosyncratic beliefs. In any case, it *does not* guarantee de facto permanent assignments. The intentionally permanent characteristic 'male', for instance, might not be de facto permanently assigned by persons who encounter trans-sexuals, that is, people who changed their sex through surgical operations.

2.3.5. We will say that for P an assignment of a characteristic Ch' to an entity x is *de facto incompatible* with an assignment of a characteristic Ch'' to the entity if and only if (a) P assigns Ch'' (Ch') de facto permanently to x and entities equivalent to x, and (b) he never assigns Ch' (Ch'') to x and entities equivalent to x.

The definition allows that de facto incompatibility is merely a habit of P, that is, it does not stipulate that P must consider Ch' and Ch'' to be mutually exclusive, although it is likely that in all interesting cases he actually does consider them to be mutually exclusive. An example will help: suppose P assigns 'human' de facto permanently to two-legged animals; suppose also that it so happens that he never assigns 'featherless' to two-legged animals: it is not likely that P then considers 'human' and 'featherless' to be mutually exclusive. If we replace 'featherless' by 'non-human' then the two characteristics are mutually exclusive and if 'human' is also intentionally permanent, P would expect to assign them de facto incompatibly. This aspect will be expressed in the next definition.

2.3.6. We will say that a characteristic Ch' is *intentionally incompatible* for P with a characteristic Ch'' if and only if (a) the characteristics are mutually exclusive for P and (b) at least one of them is intentionally permanent.

Since Ch' and Ch'' are mutually exclusive for P, he cannot conceive of assigning them at the same time to the same entity; and since the assignment of at least one of the characteristics is expected never to be revoked (it is intentionally permanent), P expects never to assign the other characteristic to the entity or entities equivalent to it. This is, in effect, P's expectation of de facto incompatible assignments of Ch' and Ch''.

In physics 'corpuscular' and 'wavelike' are intentionally incompatible characteristics (at least in Bohr's interpretation). Hence, their assignments are expected to be de facto incompatible. This expectation, however, is confirmed only in macro-physics, while it is disconfirmed in micro-physics; here, an electron 'behaves' sometimes corpuscularly and sometimes wavelike. This kind of assignment of characteristics will be defined next.[7]

2.3.7. We will say that an assignment of a characteristic Ch' to an entity x is *de facto noncompatible* for P with an assignment of a characteristic Ch'' if and only if (a) P assigns both Ch' and Ch'' to x and entities equivalent to x but (b) never at the same time.

There are many examples for this definition. Let us take, for instance, the characteristics 'magnetic' and 'not magnetic'. Both can be applied to 'a piece of iron' but never at the same time, so that their assignments are de facto noncompatible. 'Bachelor' and 'married' are usually also de facto noncompatibly assigned. But while 'magnetic/non magnetic' and 'bachelor/married' are each usually not intentionally incompatible pairs of characteristics, de facto noncompatibility of assignments can, as mentioned, also occur with intentionally incompatible characteristics, as for instance with 'male/female' with reference to trans-sexuals and 'corpuscular/wavelike' with respect to micro-objects. In these cases where characteristics are intentionally incompatible but their assignments are de facto noncompatible, there will be a psychological 'uneasiness' for P, because his expectation of de facto incompatible assignments is disconfirmed. We will elaborate this point later on when discussing an assignment paradox.

2.4. *Circumstances of Investigation*

2.4.1. Assignments of characteristics always take place under some particular *circumstances*. Many of these circumstances are recognized and specifiable, and many of these are known to be irrelevant for the assignment of a characteristic to an entity. For instance, it is irrelevant for the analysis of a blood sample under a microscope whether the microscope is painted black or gray. Yet some of the recognized and specifiable circumstances are known to be relevant.

A circumstance is 'relevant' if it is a sufficient condition for P to decide which one of two given mutually exclusive characteristics should be assigned to a particular entity. For instance, for deciding whether the characteristic 'electrically charged' or its negation should be assigned to an insulated metal sphere, one 'relevant' circumstance would be the presence of an electroscope adequately installed in its vicinity; its positive response would then lead to the assignment of 'electrically charged', its negative response would lead to the assignment of 'not electrically charged'.

It is important to distinguish from such relevant circumstances what we call 'determinative' circumstances for *P*'s assignment of a characteristic to a particular entity. They may or may not also be relevant.

A circumstance will be called *determinative* for *P*'s assignment of *Ch*′ (*Ch*″) to a particular entity if and only if (a) *P* is confronted with the mutually exclusive alternatives *Ch*′ and *Ch*″, and (b) *P* assigns *Ch*′ (*Ch*″), given the circumstance, more frequently to the entity than *Ch*″ (*Ch*′).

Let us take an example: A bubble chamber is a determinative circumstance for assigning corpuscularity to an electron because, given the bubble chamber and the alternatives 'corpuscular' and 'wavelike', an investigator will more frequently (in fact always) assign 'corpuscular' to the electron than 'wavelike'. A bubble chamber is also a relevant circumstance,[8] since it is a sufficient test condition for deciding whether 'corpuscular' or 'wavelike' should be assigned[9]. A determinative circumstance which is not 'relevant' in the above sense, is for instance the famous 'experimenter's bias', where the experimenter unwittingly influences the experiment in such a way that it will confirm his hypothesis.[10]

It is clear that determinative circumstances in our context operate only once relevant circumstances are given, since the characteristics we consider are sufficiently precise for *P* to know when he lacks a sufficient condition for assigning them (cf. 2.3.1 above). For instance, nobody will assign color characteristics in complete darkness. We will thus neglect relevant circumstances and only talk about determinative ones, assuming either that the relevant circumstances are identical with the determinative ones (as frequently in micro-physics) or that some unspecified relevant circumstances are present.

We will use '*C*' to designate a determinative circumstance (for *P* and a given *Ch* or a particular pair of characteristics).

We will also use the term 'entity per se' in this connection: An entity will be called an *entity per se* if and only if (a) the entity is undefined or (b) the entity is defined but its definition contains or implies no reference to circumstances of investigation (be they relevant or determinative or both).

For instance, an 'electron' is an entity per se; an 'electron as investigated in a bubble chamber' is not an entity per se. It is not implied that entities per se are in any sense 'simple' or 'elementary', nor do we imply any ontological difference. The distinguishing feature of an entity per se is solely the lack of reference to circumstances of investigation in the definition of its kind.

2.4.2. Suppose there exists a determinative circumstance (C) for the assignment of a characteristic to a particular entity per se. The entity per se may then be conceived as part of a complex entity whose other part is C. Following Bohr (1958, p. 64), we will call such a complex entity *phenomenon* (Ph). While for Bohr (1958, p. 39) a phenomenon in micro-physics implies certain uncontrollable interaction between the entity per se and the determinative experimental arrangements, we will not restrict the use of 'phenomenon' to situations where the term 'interaction' in this sense is applicable, or where interaction is known to be uncontrollable. 'Phenomenon' in our sense can be interpreted to be an unspecified relation between the determinative circumstance and the entity per se.

2.5. *Domains of Characteristics*

2.5.1. With respect to a given characteristic Ch', an entity x will be said to belong to *domain*$_1$ *of* Ch' for P (or, for short, to domain$_1$, when the context clearly includes Ch' and P) if and only if (a) P assigns Ch' to x per se de facto permanently, and (b) Ch' is intentionally permanent for P.

In physics, the characteristic 'corpuscular' is intentionally permanent and de facto permanently assigned to stones; thus, stones belong to domain$_1$ of the characteristic 'corpuscular' for people who take the term as defined in classical physics.

2.5.2. With respect to a given characteristic Ch', an entity will be said to belong to *domain*$_2$ *of* Ch' for P (or, for short, domain$_2$, when the context clearly includes Ch' and P) if and only if (a) P does not assign Ch' to the entity per se de facto permanently, and (b) Ch' is intentionally permanent for P.

In Bohr's interpretation of micro-physical theory, the characteristic 'corpuscular' is intentionally permanent but it is not assigned de facto permanently to electrons; thus, electrons belong to $domain_2$ of the characteristic 'corpuscular' for people who take the term as defined in classical physics but who do not assign it de facto permanently to entities per se in micro-physics.

It should be noted that reference to the intentional permanence of characteristics alone does not suffice to define the domains, because the distinction between $domain_1$ and $domain_2$ depends also on the way these characteristics are assigned by P, viz. de facto permanently or not.

2.5.3. In physics, the entities of $domain_1$ and $domain_2$ of the characteristic 'corpuscular' (or 'wavelike', for that matter) are sometimes conceived to be qualitatively different: entities of $domain_1$ are 'macro-objects' and entities of $domain_2$ are 'micro-objects'. In psychology (cf. Brody and Oppenheim, 1969) the division into $domain_1$ and $domain_2$ with respect to the characteristic 'mental' (or 'bodily') is a division, roughly, into 'naively' described entities ($domain_1$) and theoretical constructs ($domain_2$). We do not stipulate that the division into these two domains depends on, and will always yield, a division into qualitatively different entities, like 'observable-unobservable' (whatever 'observable' and 'unobservable' may mean).

2.6. *Formulation of the Paradox*

2.6.1. An *assignment paradox* arises through a clash of expected and de facto assignments of characteristics. Let Ch' and Ch'' be intentionally permanent and mutually exclusive characteristics for P (this implies they are *intentionally incompatible* as defined in 2.3.6.). If P assigns each of the characteristics de facto permanently to some different entities, $domains_1$ for both characteristics are non-empty. Let us call the *union* of these domains simply '$domain_1$ for Ch'–Ch''', and let us call the entities in it '$entities_1$'. The assignments of Ch' and Ch'' in $domain_1$ are now *de facto incompatible*. Since the characteristics Ch' and Ch'' are intentionally incompatible, de facto incompatible assignments are expected and this expectation is confirmed in $domain_1$. Suppose now that there are some entities to which P assigns Ch' but not permanently, and some entities to which P assigns Ch'' also not permanently. Clearly, these entities are in the $domains_2$ of Ch' and Ch'', respectively. Let us call the *intersection* between these domains simply '$domain_2$ of Ch'–Ch''', and let us call the entities in it 'entities$_2$'. If this intersection is non-empty, the assignments of Ch' and Ch'' in domain$_2$ are *de facto noncompatible*. Since Ch' and Ch'' are intentionally incompatible, de facto incompatible assignments are expected, but this expectation is disconfirmed in domain$_2$. This produces what we call an 'assignment paradox'.

We follow Bohr in calling this a 'paradox' for a variety of reasons. First, the disconfirmed expectation produces a feeling of uneasiness.[11] This is similar to what is called 'paradox' in other contexts.[12] The word 'paradox' itself suggests that what it refers to is 'against opinion' or 'against expectation'. Second, since at least one of the characteristics involved is intentionally permanent, the disconfirmed expectation is especially perplexing. The question, "what is it really?" is likely to express the perplexity. In physics, an assignment paradox arises through the fact that electrons sometimes 'behave' corpuscularly and sometimes wavelike, 'corpuscular' and 'wavelike' being intentionally incompatible characteristics.

2.6.2. The motive for the removal of this paradox is rarely just the uneasiness of disconfirmed expectations. In physics, for example, the assignment of 'wavelike' alone is not as important as the consequential mobilization of a theory (wave theory) which explains further observations of the electron; the uneasiness is then augmented by the fact that equivalent entities mobilize logically incompatible theories. De facto noncompatible assignments of intentionally incompatible characteristics might thus lead to strains beyond the immediate assignment paradox.[13]

2.6.3. The assignment paradox is accompanied by a *conceptual disunity*. While the former is a feeling of uneasiness and perplexity, the latter is a severe inconvenience in the use of language. Used the 'normal' way, the characteristics in question are ambiguous with respect to entities$_2$. While it is meaningful to say, 'a stone is corpuscular', the sentence 'an electron is corpuscular' has no clear meaning, at least in Bohr's interpretation of micro-physics which acknowledges the assignment paradox. The ambiguity that arises from the usual use of language with respect to entities$_2$ was one of the important obstacles Bohr wanted to overcome by his Principle of Complementarity (cf. Bohr, 1958, p. 40).

2.6.4. We can now formally define an assignment paradox. *P* is confronted with an *assignment paradox* relative to *Ch′* and *Ch″* if and only if (a) according to *P*, *Ch′* and *Ch″* are *intentionally incompatible*; (b) there are entities (entities$_1$) with respect to which the assignments of *Ch′* and *Ch″* by *P* are *de facto incompatible*; (c) there are entities (entities$_2$) with respect to which the assignments of *Ch′* and *Ch″* by *P* are *de facto noncompatible.*

3. Generalized complementarity

3.1. *Removal of the Paradox*

3.1.1. The removal of the assignment paradox might take various logically possible forms. The most obvious solution would be to abandon intentional incompatibility of the *Ch*'s. This would be analogous to abandoning a theory after disconfirming instances occurred. However, in practice this will often not be a satisfactory solution for at least two reasons: first, intentional incompatibility *is* confirmed in domain$_1$ (clause (b) in the definition of an assignment paradox). Second, intentional permanence of a characteristic, on which intentional incompatibility is built, is rarely an isolated expectation; changing it might cause a breakdown of a network of other expectations and theories for which no working alternatives are available. For example, the intentional permanence of 'wavelike' in physics is a cornerstone of wave theory for which no workable alternative exists at present. Similarly, the intentional permanence of everyday characteristics such as 'male', is based on a variety of other expectations, such as the permanence of bodily functions, sociological roles, legal and psychological identity etc.

Another solution would be to exclude domain$_2$ from the joint scope of predication of the characteristics. The price for this measure can be severely high. In physics, for instance, it would imply that we give up assigning intuitable classical characteristics (corpuscular, wavelike) to micro-objects, restricting micro-physics to the abstract formalism. For Bohr, intuitability was essential for communicating experimental results and for theorizing itself. He therefore refused to exclude domain$_2$ from the scope of predication of 'wavelike' and 'corpuscular' and suggested his Principle of Complementarity.

3.1.2. He stipulated that we *render the assignments of the characteristics* *in domain$_2$ de facto incompatible* by introducing the concept 'phenomenon' (see 2.4.2. above), in which case the conceptual unity would be obtained. In order to do this, however, the determinative circumstances of assignments (C's) for intentionally incompatible characteristics must be pairwise mutually exclusive. This means that there is no C under which both Ch' and Ch'' can be assigned to an entity of domain$_2$ of Ch'–Ch''. Since a phenomenon contains only one C, mutually exclusive C's must be part of mutually exclusive phenomena. Assignments of intentionally incompatible characteristics to these phenomena can now be de facto incompatible.

In micro-physics 'corpuscular' and 'wavelike' are intentionally incompatible, but they can only be de facto noncompatibly assigned to the electron per se. The determinative experimental arrangements under which 'corpuscular' is assigned, however, are pairwise mutually exclusive with the determinative experimental arrangements under which 'wavelike' is assigned. 'Combining' the electron with the experimental arrangements of its investigation to form a 'phenomenon' (e.g. 'electron as investigated in a bubble chamber') will allow assignments of 'corpuscular' and 'wavelike' to mutually exclusive phenomena. De facto incompatible assignments of the characteristics are thus possible also in micro-physics. To call an electron *per se* corpuscular or wavelike is for Bohr elliptical and meaningless; but unambiguous use of the characteristics with regard to entities$_2$ is possible after the Ch's are assigned to phenomena instead of entities$_2$ per se. Conceptual unity in the joint scope of predication of Ch'–Ch'' now obtains.

3.1.3. We can now *formally define complementarity*: Given Ch' and Ch'', and given C' and C'', then the two phenomena Ph' and Ph'' stand in a relation of *complementarity* for P if and only if the following conditions obtain: (a) P is confronted with an assignment paradox relative to Ch' and Ch'' if he assigns them to entities$_2$ per se; (b) assignments of Ch' and Ch'' by P to entities$_2$ depend on pairwise mutually exclusive determinative circumstances, C' and C'';[14] (c) the assignment paradox is removed by assigning Ch' and Ch'' to Ph' and Ph'' respectively and not to entities$_2$ per se.

3.2. *Comment*

3.2.1. The reader will have noted that the complementarity relation is defined to hold between *phenomena* although some previous applications defined it as a relation between characteristics. There is no inconsistency involved, solely a shift in emphasis which parallels Bohr's own development of this principle. Since a phenomenon contains a *C*, and since *C* is defined with respect to the assignments of a particular *Ch*, complementarity could be defined either between *Ch*'s or between *Ph*'s without any factual difference. However, in Bohr's own writing, determinative experimental arrangements and phenomena became more and more important, so that he himself shifted to call phenomena complementary rather than characteristics.[15] We therefore follow this change in emphasis.

4. Epistemological dilemmas

4.1. *The Intra-Domain Dilemma*

4.1.1. Failure to see that the complementarity relation comprises these main elements, viz. assignment paradox based on two domains and dependence on determinative circumstances in domain_2, has led to quite a number of *semantic misunderstandings*. Many authors, applying the Principle of Complementarity to fields other than physics, have interpreted this principle to be a device for reconciliation of various irreconcilable approaches. Since we believe that Bohr's principle does not lend itself to such uses, it is worthwhile to try to uncover the source of such misunderstandings.

In particular, we think that these authors have in mind a certain epistemological dilemma which they (wrongly) *hope* to solve through 'complementarity', leaving out another epistemological dilemma which they *could* solve through complementarity. Both dilemmas are closely related to the assignment paradox and we will attempt to define them.

4.1.2. We will say that *P* is in an *epistemological dilemma* relative to two alternatives (whatever they are) if and only if (a) the alternatives are mutually exclusive for *P*, (b) *P* needs epistemological criteria for choosing among them, and (c) *P* has no such criteria.

4.1.3. If two mutually exclusive characteristics, *Ch'* and *Ch''*, create an assignment paradox for *P* when assigned to entities_2 per se, and if their assignments to entities_2 depend on pairwise mutually exclusive determinative circumstances, C' and C'', then P is in an epistemological dilemma relative to C' and C''.

In physics, for instance, the electron as investigated in a bubble chamber 'behaves' corpuscularly; as investigated by a nickel crystal, it 'behaves' wavelike. If P has epistemological criteria for discounting either C' or C'' as leading to an 'illegitimate' assignment, or if we have epistemological criteria for rendering the choice unnecessary (by for instance making the characteristics assigned under C' and C'' compatible), we would have solved the dilemma. However, there are (for Bohr, at least) no epistemological criteria for preferring the bubble chamber (a C') over a nickel crystal (a C'') for investigating the behavior of the electron. No description of its behavior could be considered complete without its investigation under C' conditions as well as under C'' conditions. At least in Bohr's interpretation of micro-physics, it is also impossible to render the choice unnecessary.

Since the C's play their crucial role for us only *within domain*$_2$, we will call this epistemological dilemma an *intra-domain dilemma*.

4.1.4. When the C's are combined with entities$_2$ per se to form complex entities, viz. phenomena, the de facto noncompatibility of the assignments of the *Ch*'s to entities$_2$ per se turns into de facto incompatibility of assignments to phenomena and the assignment paradox is removed. The intra-domain dilemma, however, remains because in choosing C's we choose phenomena, and complementarity does not supply P with any epistemological criteria for choosing between mutually exclusive C's. Falsely to call an intra-domain dilemma 'complementarity' does not take the bite out of the dilemma; it rather confuses the issue. This is exactly what happens frequently in the literature about complementarity. Likening 'approaches', 'standpoints', 'views', etc. to determinative circumstances of investigation (C's), authors have erroneously used the term 'complementarity' with respect to intra-domain dilemmas. William Pollard (1958), for example, cites the mechanistic and vitalistic approach in psychology as being complementary in Bohr's sense. Pollard also mentions "the paradox of freedom and providence" as an example of complementarity and he states: When "two approaches represent mutually exclusive *modes* of explanation (their) equal validity places them in a complementary relationship to each other" (p. 150). To claim that approaches are equally valid is to claim that one has no epistemological criteria for preferring one over the other; together with mutual exclusiveness of approaches, equal validity establishes, in effect, an intra-domain dilemma. The mere label 'complementarity' does not solve this dilemma.

4.2. *The Inter-Domain Dilemma*

4.2.1. Important for grasping the function of complementarity is an epistemological dilemma Pollard does not state, namely the dilemma between discarding and keeping entities which render the use of characteristics ambiguous. Since this dilemma is identical with including domain_2 in or excluding it from the scope of predication of a *Ch* (or pair of *Ch*'s), we will call it *inter-domain dilemma* between the two domains.

Bohr, for instance, was confronted with the inter-domain dilemma where he could either refrain from using classical concepts in micro-physics, preserving their unambiguous use but rendering micro-physics unintuitable, or applying classical concepts to micro-objects, gaining intuitability at the price of ambiguous use of the concepts (Bohr, 1958, p. 39f.). The Principle of Complementarity was designed with regard to the solution of this dilemma. It allows application of classical concepts in micro-physics without rendering them ambiguous. This solution entails, however, that the *Ch*'s are applied in an *unusual* way in micro-physics, namely, to phenomena instead of to entities per se.[16]

4.3. *Dilemmas and Complementarity*

4.3.1. Introducing epistemological dilemmas in *two* dimensions has great systematic power. Thereby, Pollard's (and others') source of misinterpretation of complementarity can be easily located. Pollard tried to solve the *intra*-domain dilemma with complementarity, although it cannot be done; and he did not realize that this principle does indeed solve the *inter*-domain dilemma. The first is based on a *semantic misunderstanding*, the second stems from the *omission* of domain_1. But Pollard is only one among many who make similar mistakes.[17] The reason for citing him rather than others was solely the fact that he is not only a theologian but also a trained physicist who can be expected to know the difficulties of generalizing from physics to other fields.

4.3.2. It might be concluded that it is a shortcoming of complementarity not to solve also the intra-domain dilemma. However, this conclusion would be seriously mistaken. Rather, *insolubility* of the intra-domain dilemma is *crucial* for any meaningful application of the Principle of Complementarity.

Suppose the intra-domain dilemma could be solved; then we would have epistemological criteria for choosing between the *C*'s or making the choice unnecessary. In physics, for instance, the conflicting findings under mutually exclusive *C*'s were reconciled by various theorists in various ways. It was suggested that the electron is wavelike but not corpuscular and that particles are only more or less temporary entities with the wave field (Schrödinger, 1953); or it was suggested that the electron is corpuscular but not wavelike and that 'waves' are only waves of probabilities (Born, 1949); or it was suggested that electrons are both wavelike and corpuscular (DeBroglie, 1939); or neither wavelike nor corpuscular (Margenau, 1950). For reasons beyond the scope of this paper,[18] Bohr rejected all of these 'solutions' to the intra-domain dilemma and indeed stipulated that this dilemma is insoluble (Bohr, 1958, p. 62). Only under this condition is the assignment of characteristics to phenomena rather than to entities per se justified and necessary.

4.4. *Justified Application of Complementarity*

4.4.1. While for Bohr the intra-domain dilemma was an integral part of complementarity, we defined complementarity independently of this dilemma in order to explicate its formal structure. We thus have to distinguish between two cases: first, the application of complementarity is *possible*; second, the application of complementarity is *justified*. In the first case, many trivial applications are likely to occur, reducing complementarity to a formal exercise;[19] only the second case is theoretically interesting. We will now define the conditions for each:

4.4.2. The application of the Principle of Complementarity to a field is *possible* for *P* if and only if all conditions of its definition are met for *P* (cf. 3.1.3.).

4.4.3. The application of the Principle of Complementarity to a field is *justified* for *P* if and only if (a) its application is possible for *P*, (b) there is a related intra-domain dilemma for *P*, (c) *P* accepts a theory that precludes the solution of the intra-domain dilemma and/or (d) *P* does not accept any theory from which such a solution follows.

4.4.4. With regard to any non-trivial application of the Principle to a particular field, *a particular insoluble intra-domain dilemma in this field will have to be made explicit first*. This requires detailed justifications for the alleged insolubility of the intra-domain dilemma (for an example, see Feyerabend 1968 and 1969). With this requirement, complementarity will aid the growth of knowledge rather than dogmatize feeble theories.

This paper may thus help to prevent the indiscriminate use of the Principle of Complementarity in fields where no intra-domain dilemma can be or has been defined.

5. Discussion

5.1. We hope to have shown in sufficient detail that complementarity, as understood by Bohr, can successfully be generalized. This generalization has several *novel* aspects.

5.1.1. One of these novel aspects of our general definition is that we made explicit what problems the Principle of Complementarity solves: (a) the assignment paradox and the related mobilization of incompatible theories, (b) the *inter*-domain dilemma and the related conceptual disunity.

The elements that served to define the assignment paradox are novel, too. Instead of dealing just with incompatibility and noncompatibility of characteristics, we introduced intentional incompatibility of characteristics and de facto incompatibility and noncompatibility of assignments. This distinction, which proved to be fruitful in our context, may find some interesting applications in other contexts as well, such as psychology and the theory of concept formation.

5.1.2. Our general definition of complementarity admits in principle *any* pair of characteristics, as long as it satisfies the formal conditions for *P*. This might lead to trivial applications of this principle. We therefore formulated conditions under which we think the application justified. This split between the formal Principle and its conditions for justified application is also novel. It is based on the separate definitions of complementarity and the intra-domain dilemma. Insolubility of the latter is a necessary condition for the justified application of the former.

5.2. Generalized complementarity, as we defined it, is of course related to complementarity in the field of physics. It might be interesting to see what *direct* correspondences between the two definitions obtain.

5.2.1. In our view, the following two *formal* correspondences are noteworthy: first, both in physics and in the general definition, *disjoint classes of entities* (domains) can be distinguished. In physics, they are represented by the classes of macro- and micro-objects; in the general definition, correspondingly, they are represented by the classes of entities_1 and entities_2. Second, there are two *disjoint classes of circumstances* under which entities in domain_2 are investigated. In physics, they are represented by experimental arrangements; in the general definition, they are represented by determinative circumstances.

5.2.2. In addition, there exist the following *epistemic* correspondences between the two definitions: first, one must admit the possibility that, with the progress of physics, the Principle of Complementarity may cease to be applicable to this field; correspondingly, our generalization of Bohr's principle is, in its applicability, relativized to *P*'s present body of knowledge (regarding the insolubility of the intra-domain dilemma). Second, Bohr does not answer the question whether micro-objects 'really' are corpuscular or wavelike (whatever the term 'really' may mean), and he renders this question meaningless in micro-physics. The general definition also acknowledges that accepting complementarity in a nontrivial way commits to this 'epistemic neutrality' with regard to entities_2 in whatever field the principle may be applied.

5.3. These direct correspondences between the two definitions must be qualified in various respects.

5.3.1. In physics, entities_1 (macro-objects) and entities_2 (micro-objects) can be conceived to be qualitatively different. By contrast, the general definition does not stipulate *any* restrictions regarding entities; therefore, qualitative differences between entities_1 and entities_2 are also not required.

This difference may be important for those who believe that complementarity is restricted to situations where characteristics for one kind of objects are extended to cover another kind of objects. According to our general definition, entities_1 and entities_2 may belong to the same kind and differ only with respect to the de facto assignment of characteristics.

5.3.2. While the definitions of noncompatibility and incompatibility in physics refer to dispositional properties, the general definition is not limited to such characteristics. As long as characteristics are intentionally permanent, they may appear in a complementarity relation, even if they are, say, directly observable characteristics. Thus, intentional permanence is less restrictive than dispositionality, and many dispositional characteristics are not intentionally permanent (e.g. 'magnetic', cf. Hempel, 1952, p. 24f).

5.3.3. In physics, justification of complementarity rests on the assumption of an uncontrollable interaction between the electron per se and the experimental arrangement used to investigate the electron. By contrast, the justified application of the general definition requires generally an insoluble intra-domain dilemma, whether this dilemma is based on uncontrollable interaction or not.

5.3.4. In physics, the assignment paradox is produced through more or less intersubjective bodies of theories and has thus an intersubjective character itself. The general definition is compatible with a 'subjective' assignment paradox in the sense that intentional permanence and incompatibility might not follow from any intersubjectively testable theory. It is for instance conceivable that complementarity may be applicable in the field of law where intentional permanence and incompatibility rest on tradition or 'experience' rather than on any testable theory.

6. Conclusion

6.1. Representatives of various fields may not be satisfied by the mere fact that complementarity can be generalized and thereby possibly applied in novel contexts. They want to know what results of these possible applications are relevant for them, if any.

We believe that such results exist. Among several ways to summarize them, we choose one which accentuates considerations important for the choice of a complementarist's solution to the problems described.

6.2. Where language (and 'models' or 'pictures' conveyed by it) fails persistently in describing certain entities adequately, we can choose to give up using it in this descriptive function. In micro-physics, for instance, we can resign to completely formal means, viz. the abstract formalism. However, this measure carries a high price, since it destroys intuitability and severely impedes the effectiveness of communication.

If, in the face of this price, we choose intuitability (to which Bohr ascribed utmost importance for science itself), that is, if we choose to use intuitable language despite its inadequacy, then complementarity has advantages over noncomplementarity (for instance the solution of the inter-domain dilemma). But in order to make a meaningful choice between complementarity and noncomplementarity (given the insolubility of the intra-domain dilemma), it should be born in mind that *both* accepting and not accepting complementarity involve certain 'prices' to be paid for certain 'advantages'.

6.2.1. If *P* accepts complementarity, he is confronted with at least the following 'bargain-situations':[20]

(a) for the advantage of having the *assignment paradox removed*, he pays the price of conceiving the question whether an entity_2 is 'really' *Ch′* or *Ch″* as *meaningless*;

(b) for the advantage of *conceptual unity* he pays the price of tolerating *unusual use of language.*

6.2.2. Conversely, if *P* does not accept complementarity, he is confronted with at least the following bargain-situations:

(a) for the advantage of *not* conceiving the question whether an entity is 'really' *Ch′* or *Ch″* as *meaningless*, he pays the price of *remaining in an assignment paradox*;

(b) for the advantage of retaining *usual use of language*, he pays the price of *conceptual disunity.*

6.3. The advantages of complementarity force *P* to forego the advantages of noncomplementarity and vice versa, because the advantages of one are the prices of the other. Given such situations, it is only too human to desire the impossible, that is, to have *both* advantages although one is only attainable at the price of the other.

We think that the impossibility of avoiding bargain-situations in epistemology is one of the most important insights that can be gained from this discussion of complementarity.[21]

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NOTES

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1 Bohr first introduced his Principle of Complementarity in his lecture 'The Quantum Postulate and the Recent Development of Atomic Theory', reprinted in Bohr (1934). Our account of complementarity in physics in the present paper will serve only illustrative purposes and will be more than simplified. For a detailed discussion of complementarity and its relevance in physics today see Feyerabend (1968) and Feyerabend (1969); for an explicit reconstruction of the principle in physics, see Bedau and Oppenheim (1961).

2 Cf. Bohr (1958), p. 76ff. *et passim.*

3 Similarly, in biology, with reference to the correspondence (homology) of, say, a human's arm and a bird's wing, a *general definition* of the concept of 'Bauplan' is presupposed under which the Bauplan of humans and of birds are subsumed. See Woodger (1945, p. 109). See also Rescher and Oppenheim (1955, p. 90ff.).

4 Compare Hempel (1939) and 'codability' discussed in Brown and Lenneberg (1954).

5 The concept 'quasi-psychological', as distinguished from 'psychological' and 'purely logical', is introduced by Carnap (1962), and referred to a completely rational person. We, however, do not assume that the person must be rational beyond the assumptions made in the text.

6 The term 'de facto permanent' as also any other related term we define will be used in whatever grammatical form seems convenient. Thus, we will use 'de facto permanence', 'de facto permanently', etc.

7 There are certain logically possible relations which we will not need and which we therefore do not define; for instance, 'intentional noncompatibility' of *Ch*'s, or 'de facto compatibility' of assignments, and 'intentional compatibility' of *Ch*'s. These relations could be easily defined on the basis of the present definitions. The fact that we omit them should not indicate that we believe they could not be usefully employed in other contexts; on the contrary, we believe that the general distinction between the intended relationship of characteristics and the de facto relationship of their assignments will prove to be useful in other contexts as well.

The term 'intentional' is not used here in the technical sense of Husserl's phenomenology.

[8] Since this coincidence of determinative and relevant circumstances is typical in micro-physics, this distinction is often not made explicit.

[9] The fact that physicists today would use the bubble chamber to find out other characteristics of the behavior of electrons or other micro-objects is beside the point. The bubble chamber is still a relevant and determinative circumstance for assigning the characteristic corpuscular to micro-objects.

[10] Cf. Rosenthal (1963); determinative circumstances are, generally speaking, annoying if they are identical with relevant circumstances or if they are hard to notice or cannot be neutralized in some way. Complementarity is closely connected to such annoying situations.

[11] For a psychological discussion of the relation of conceptual conflicts to epistemic behavior see Berlyne (1960). Carlsmith and Aronson (1963), among others, show that disconfirmed expectations lead to 'uneasiness' (in this case 'cognitive dissonance').

[12] Quine (1966) introduced various distinctions between kinds of paradoxes. The 'veridical' paradox establishes a truth which at first blush seemed perplexing; the 'falsidical' paradox is a perplexing falsity which at first blush seemed true; antinomies are paradoxes which can be removed "by nothing less than a repudiation of part of our conceptual heritage" (p. 11). Taken this broad definition of antinomies, the assignment paradox is indeed an antinomy (cf. 3.1f.) although it is very different from, say, Russell's antinomy.

[13] Heisenberg's Uncertainty Principle is often viewed as the foundation of complementarity (see for instance Bedau and Oppenheim, 1961, p. 204). For this reason, the question may arise whether Heisenberg's principle implies an assignment paradox even in our general formulation. We believe that this question can be answered in the affirmative.

The crucial elements in our general definition are intentional permanence of characteristics and de facto permanence of assignments. Keeping this in mind, we can express the assignment paradox of position/momentum in the following way: 'has an exactly ascertainable position', or, for short, 'q', is an intentionally permanent characteristic; thus we expect to assign it de facto permanently to everything in its scope of predication. The characteristic q is now (trivially) *intentionally incompatible* with $-q$ ('has not an exactly ascertainable position'), so that *de facto incompatible* assignments are *expected.* In macro-physics, this expectation is confirmed; in micro-physics this expectation is *disconfirmed* because q and $-q$ can only be *de facto noncompatibly* assigned to electrons, that is, electrons sometimes have an exactly ascertainable position and sometimes not. This meets *all* of our requirements for an assignment paradox. The reader can analogously construct a paradox with the characteristic 'has an exactly ascertainable momentum', or, for short, 'p', and $-p$. Thus, Heisenberg's principle implies two assignment paradoxes. In addition, the two paradoxes are related by the fact that, according to Heisenberg's principle, assignment of p is a determinative circumstance of assigning $-q$, and assignment of q is a determinative circumstance of assigning $-p$. The attempt to construct one assignment paradox on the basis of p and q must fail, because p and q are intentionally *compatible* in macro-physics and therefore do not fit the general definition.

[14] We do by no means stipulate that all assignments in domain_1 are independent of C's. However, we will not be concerned with this dependence unless it leads to de facto noncompatible assignments (which, by definition, occur only in domain_2).

In other words, dependence on C's is *not* required in domain_1, whereas in domain_2 it *is* required.

[15] See Petersen (1968, p. 126f.).
[16] In order to prevent misunderstandings, we would like to add that the term 'unusual use of language' is used in two different ways; while we mean by it the assignment of *Ch*'s to phenomena instead of to entities per se, physicists mean by it the assignment of classical physical concepts to micro-objects.
[17] For example Barbour (1969, p. 214), Margenau (1960, p. 114), Pauli (1950), Frank (1957), Bunge (1955). Even in Bohr (1958, p. 92, *passim*) and in Heisenberg (1958, p. 179) one finds occasionally statements in which no clear distinction between complementarity and what we call intra-domain dilemma is made.

There is another kind of semantical misunderstanding of complementarity which arises through the omission of the requirement that the characteristics must be mutually exclusive. For example, Coulson speaks of "parallel interpretations" as being complementary, and he means *compatible* but unrelated accounts of the same entity (like various descriptions of a building). See Coulson (1955, pp. 66ff.).
[18] For a detailed discussion of critiques of complementarity and Bohr's defense, see Feyerabend (1968) and (1969).
[19] Cf. Hutten (1956, p. 191).
[20] Cf. Lindenberg and Oppenheim, 'The Bargain Principle' (forthcoming) in which 'bargain-situations' are discussed in detail.
[21] As to the relevance of bargain-situations in epistemology, see Weyl (1949, p. 116, *passim*).

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